Efficiency of Solid State Photomultipliers in Photon Number Resolution

S. Vinogradov, T. Vinogradova, V. Shubin, D. Shushakov, and K. Sitarsky

***Abstract*—Solid State Photomultipliers (SSPM) are widely recognized as new generation of photodetectors competitive with APD and PMT in various applications. SSPM advantages are high gain and ultra-low excess noise factor of internal amplification resulting in ability to detect single photons, high photon detection efficiency, fast response and good time resolution. SSPM drawbacks are high dark count rate, high probability of cross-talk and afterpulsing, and low dynamic range.**

Many applications in Nuclear Science and Medical Imaging for photodetector with scintillator require good energy resolution, which is represented with respect to photodetector itself by pulse height resolution or so-called Photon Number Resolution (PNR).

The purpose of this study is to express SSPM PNR in analytical form taking into account excess noise factor of cross-talk and afterpulsing as well as nonlinearity of photoresponse due to limited number of pixels and finite reset time. Normalization of PNR relatively to ideal detector characterizes the intrinsic detector performance in terms of total excess noise factor of photodetection or, inversely, in terms of detective quantum efficiency, which seems to be very powerful tool for SSPM optimization and evaluation of SSPM applicability and competitiveness.

Index Terms—Detective Quantum Efficiency, Excess Noise Factor, Probability Distribution, Solid State Photomultipliers.

I. Introduction

Use of overcritical avalanche process with negative feedback (limited Geiger mode) for proportional detection of an extra-low light signal was first proposed more than 10 years ago [1]-[5]. Limited Geiger mode avalanche multiplication is characterized by high gain (10^4 – 10^6) and ultra-low excess noise factor (1.01 – 1.05) due to negative feedback that is responsible for the quenching of breakdown with efficient suppression of output charge fluctuations. Photodetectors utilizing this mode are known now as Solid State Photomultiplier (SSPM), Silicon Photomultiplier, Geiger mode APD matrix, Multi-Pixel Photon Counter, and a few more names. Let us therefore use the collective name SSPM for this generation of photodetectors. To perform proportional detection of light pulses, these devices are typically designed as matrix of APD pixels operating in limited Geiger mode with common output. Multi-pixel architecture with low noise high gain multiplication of each pixel is the base for the SSPM advantage in a few photon pulse detection, but on the other hand it is the source of SSPM drawbacks and trade-offs. High gain results in considerable excess noise of cross-talk and afterpulsing processes. Limited number of pixels and its finite reset time are the sources of non-linearity and saturation of SSPM photoresponse in detection of short (relatively to reset time) and long light pulses correspondingly. Higher PDE requires higher fill factor (sensitive area to total area fraction), i.e. lower density of pixels, thus sensitivity contradicts with linearity.

Every specific application of SSPM requires at least preliminary analysis of its applicability and competitiveness with PMT, APD or other SSPM to balance, for example, sensitivity and linearity, and then, at evaluation stage, – optimal selection of operating voltage to balance, for example, photon detection efficiency, dark count rate, cross-talk, and afterpulsing to achieve the best performance.

Monte Carlo simulations of stochastic processes of photodetection in SSPM including cross-talk, afterpulsing and saturation effects became popular and powerful tool in the last years [6]-[9], but they can not substitute needs in analytical model.

The goal of this paper is to find the analytical expressions clarifying the influence of a few key SSPM parameters on PNR that are useful for such analysis and optimization.

Let us distinguish PNR as a measure of resolution of output signal calibrated into equivalent input scale of number of photons (equivalent input resolution) from such terms as pulse height resolution (raw output signal resolution), intrinsic energy resolution (intrinsic detector equivalent resolution calibrated in energy scale in dependence on specific scintillator-photodetector calibration curve) and so on.

Our consideration is limited by detection of light pulses with Poisson statistics of incident number of photons producing output charge in some time gate, and resolution is expressed in standard deviation unit rater then in FWHM of probability density function because probability distribution of output signal may be non-Gaussian, its exact form may be unknown, so FWHM approach may not be applicable in these cases.

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II. KEY FACTORS OF SSPM RESOLUTION

A. Vacuum PMT resolution model

Let us start with well-known model of PMT resolution expressed in units of photoelectrons (defined as number of electrons in output charge to gain ratio) and inversely equal to Signal-to-Noise Ratio (SNR) [10]:

$$RES = \frac{1}{SNR} = \frac{\sigma_{s+n}}{\mu_s} \quad (1)$$

Where

$$\mu_s = Npe = PDE \cdot Nph \quad (2)$$

is mean output signal;

Npe is mean number of photoelectrons,

Nph is mean number of incident photons,

PDE is Photon Detection Efficiency,

$$\sigma_{s+n} = \sqrt{[Npe + Nd \cdot (1 + 1/j)] \cdot Fm} \quad (3)$$

is standard deviation of output signal with noise;

$$Nd = DCR \cdot Tgate \quad (4)$$

is dark noise

DCR is Dark Count Rate

$Tgate$ is detection time or time gate

j is number of independent measurements of DCR (in further consideration let us assume $j \to \infty$)

$$Fm = 1 + \frac{\sigma^2(gain)}{\mu^2(gain)} \quad (5)$$

is excess noise factor of random multiplication process with mean gain $\mu(gain)$.

Thus,

$$RES = \sqrt{\frac{Fm}{Npe}\left(1 + \frac{Nd}{Npe}\right)} \quad (6)$$

Expression (6) assumes negligible electronic noise component due to high gain of PMT. Non-linearity of output signal is not considered as well as afterpulsing.

Thus, we can point out only three intrinsic parameters of PMT as key factors determining resolution at given Nph :

1) Characteristic of single photon detection – PDE
2) Characteristic of single electron multiplication – Fm
3) Characteristic of dark noise – DCR.

B. Specific key factors affecting SSPM Resolution

In sense of photodetection statistics SSPM behavior is similar to that of PMT in general. Single photon detection and dark noise statistics are exactly the same; multiplication mechanisms are rather different, but excess noise factor approach may be adequately used in the same way for both.

However, quantitative difference in DCR is significant, especially for SSPM operating at room temperature (10^5 - 10^6 cps/mm^2) but correct estimation of dark noise term with Poisson statistics seems to be obvious.

Situation with excess noise of avalanche multiplication is not so clear. Ultra-low excess noise factor Fm ~ 1.01 – 1.05 of Geiger mode avalanche limited by negative feedback (quenching) was observed in many reports for SSPM of different types. It was measured in correspondence with (5) applied in the most cases to single electron peak of charge / gain / pulse height distribution histogram [11]. Second and higher peaks of the histogram are also very narrow and show about the same value of $\sigma(gain)$ as the first one. On the other hand if the total histogram distribution is represented in term of excess noise factor it appears to be much higher (1.1 - 1.3 and more). This fact is recognized to be caused by optical cross-talk, which produces the chains of the secondary pulses in the same time with the initial primary pulse. Afterpulsing also has the same statistical nature, and it also adds excess noise in number of counts or in charge integrated in some time gate. While we consider detection of light pulse by using charge integration, we can not distinguish cross-talk and afterpulsing events. Let us further call any and both of them jointly as duplications. Thus, excess noise of duplications seems to be the same or even more important factor for SSPM as excess noise of dynode cascade multiplication for PMT (1.1 – 1.3).

Another well-known specific issue of SSPM is essential non-linearity due to limited dynamic range of output response. Two kinds of output response saturation should be taken into account:

Output signal amplitude or charge saturation due to limited number of pixels ($Npix$) in case of short light pulse detection,

Output signal count rate saturation due to limited number of recovered pixels ($Npix / Treset$) during long light pulse detection; $Treset$ is reset time of a single pixel, and short and long pulse times are defined in comparison with its value.

So, duplications and non-linearity are important factors affecting SSPM performance, and Poisson behavior of SSPM signals is questionable especially near saturation.

III. EXCESS NOISE FACTOR OF DUPLICATIONS

Excess noise factor (ENF) for amplification of a noisy signal is defined as the squared SNR degradation rate from input to output of the detector:

$$ENF = \left(\frac{SNR\_in}{SNR\_out}\right)^2 \quad (7)$$

Our model of duplication process caused by primary Poisson events describes compound Poisson distribution of total number of output events (including primary ones as input events) [12].

Accordingly with the model results

$$
\begin{aligned}
\mu_{comp} &= \frac{Npe}{1 - Pdup} \\
\sigma^2_{comp} &= \frac{Npe \cdot (1 + Pdup)}{(1 - Pdup)^2} \\
Fdup &= 1 + Pdup \\
RES &= \sqrt{\frac{Fdup}{Npe}}
\end{aligned}
\tag{8}
$$

Where

$Pdup$ is probability of at least one duplication event,

$Fdup$ is excess noise factor of duplications.

Compound Poisson distribution examples are presented in Fig. 1 to show how increase of duplication probability results in increasing of mean and width of distribution, i.e. degradation of SSPM resolution (8).

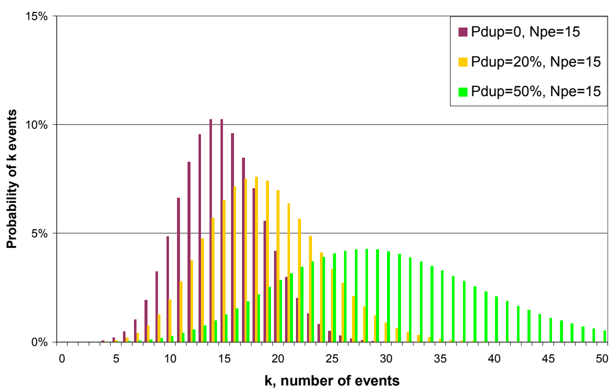

Fig. 1. Compound Poisson distribution function for Npe=15, Pdup = 0, 20%, and 50%; (Pdup=0 is pure Poisson law).

IV. Non-Linear Resolution

A. Non-linear approach in general

Using a detector to convert input signal into output one in order to resolve different levels of input signal, say, number of incident photons, we often assume linearity of output signal and rely on resolution of output signal as equal measure for resolution of input signal.

This approach is widely used for PMT, and resolution is expressed in correspondence with (1) applied to output signal and noise of PMT. In case of SSPM output signal is determined by mean number of triggered pixels Ns, which is equal to *Npe* at low signal level and tends to saturation at high signal level.

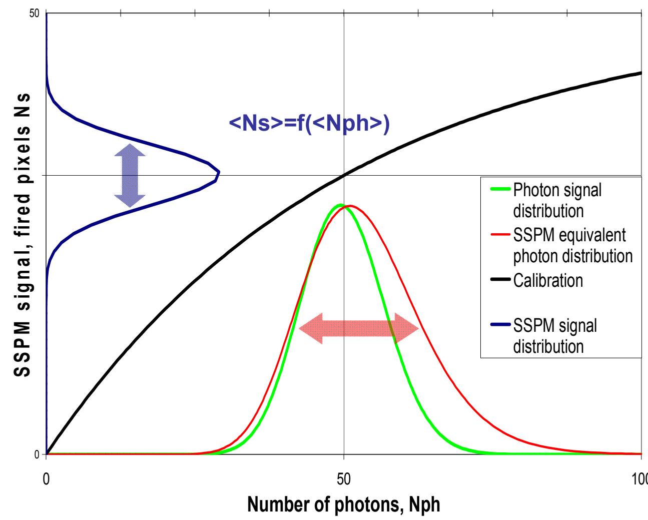

Fig. 2. Photon Number Resolution schematics corresponding to binomial non-linearity SSPM @Nph=50, Npix=50, PDE=100%, Fm=Fdup=1, Nd=0.

If output *Ns(Nph)* is non-linear, we should distinguish between resolution of equivalent photon distribution of output signal translated into input signal scale as *PNR* and resolution of output signal itself is *RESs*. This situation is illustrated on Fig. 2, where the sole source of *PNR* degradation is non-linearity. Thus, in general case we should use expression:

$$
\begin{aligned}
PNR &= RES_s \frac{Ns(Nph)/Nph}{dNs(Nph)/dNph} \\
RES_s &= \frac{\sigma(Ns)}{Ns}
\end{aligned}
\tag{9}
$$

Why and when non-linear approach is required? For example, in saturation when output signal $Ns \sim const$, deviation of signal $\sigma(Ns) \sim 0$ (because input signal of any *Nph* in the range above saturation produces fixed output *Ns*), and $RESs \sim 0$ what is senseless (better then for ideal detection $1/\sqrt{Nph}$). Expression (9) allows us to avoid this problem, and it is the ground for our analysis of SSPM efficiency in *PNR*.

B. SSPM Resolution of Short Light Pulse

Non-linearity and saturation of SSPM response in case of short multi-photon pulse detection is evident; it was observed long ago [11] and corresponding expression for mean number of triggered pixels was presented. Simulation of SSPM response was reported in [13] including behavior of standard deviation of *Ns*: its initial rise at low *Nph* and fall at saturation was shown.

Complete analytical model of SSPM response as random signals with binomial probability distribution was reported in [14] including its representation in form of equivalent photon distribution. Results of this analytical model were supported by Monte-Carlo simulation.

The same analytical results for *PNR* in slightly different terms was presented in [15] and supported by experimental data (see Fig. 3).

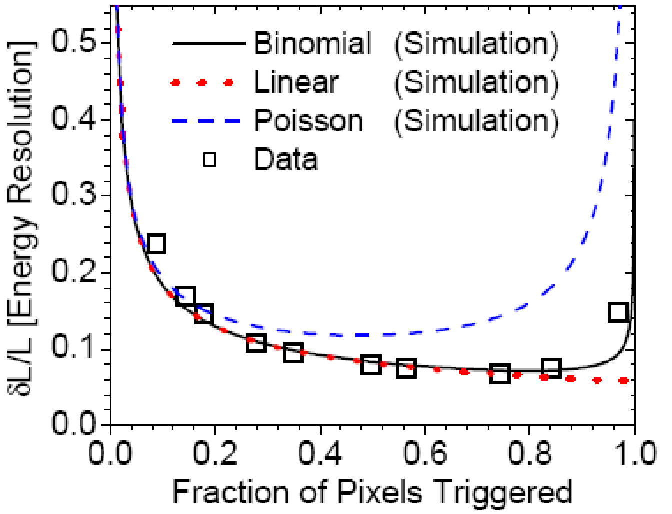

Fig. 3. Intrinsic Energy Resolution of SSPM for binomial distribution model from [15].

So, if light pulse duration is much less then reset time of SSPM pixel, output signal follows binomial distribution and its mean and variance may be expressed as (rewriting expressions from [14] and [15] into functions dependent on *Nph*):

$$Ns = Npix \cdot \left[1 - \exp(-\frac{Nph \cdot PDE}{Npix}) \right] \tag{10}$$

$$\sigma^2(Ns) = Npix \cdot \left[1 - \exp(-\frac{Nph \cdot PDE}{Npix}) \right] \cdot \exp(-\frac{Nph \cdot PDE}{Npix}) \tag{11}$$

Applying (10) and (11) to (9):

$$PNR = \sqrt{\frac{Fnl}{Nph \cdot PDE}} \tag{12}$$

Where

$$Fnl = \frac{Npix \cdot \left[\exp\left(\frac{Nph \cdot PDE}{Npix} \right) - 1 \right]}{Nph \cdot PDE} \tag{13}$$

C. SSPM Resolution of Long Light Pulse

If light pulse duration *Tpulse* exceeds pixel reset time *Treset* then number of output pulses increases in comparison with binomial distribution case due to repetitive recovering and retriggering of pixels. This situation was observed in many studies and in a few reports some analytical expressions for SSPM response were presented [16], [17]. These expressions describes only mean output signal *Ns*, and without information on *σ(Ns)* it is insufficient to deal with its resolution.

For our analysis of SSPM resolution in this case we turn to grounds of probability theory applied to non-paralysible Geiger counter with dead time and incident Poisson flux [18]. Probability distribution of number of output counts in this case approaches to Gaussian, and mean number of counts is well-known dependence of measured counts on true number of counts (dead-time correction factor).

Assuming array of pixels as *Npix* independent Geiger counters output signal may be expressed as:

$$Ns = \frac{Nph \cdot PDE}{1 + \frac{Nph \cdot PDE}{Npix} \cdot \frac{Tres}{Tpulse}} \tag{14}$$

$$\sigma^2(Ns) = \frac{Nph \cdot PDE}{\left(1 + \frac{Nph \cdot PDE}{Npix} \cdot \frac{Tres}{Tpulse} \right)^3} \tag{15}$$

Applying (14) and (15) to (9):

$$PNR = \sqrt{\frac{Fnl}{Nph \cdot PDE}} \tag{16}$$

where

$$Fnl = 1 + \frac{Nph \cdot PDE}{Npix} \cdot \frac{Tres}{Tpulse} \tag{17}$$

Thus, if *Tpulse* becomes longer *Treset* and more then non-linearity decreases significantly compared with binomial case as demonstrated on Fig. 4.

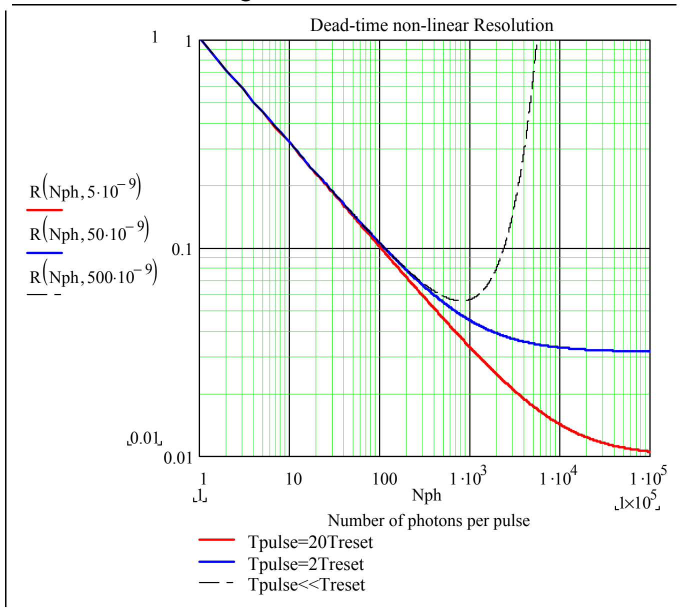

Fig. 4 . Photon Number Resolution of SSPM in case of variable reset time:
black dotted line – Tpulse << Treset (binomial distribution)
blue solid line – Tpulse =2 Treset (dead-time Gaussian distribution)
red solid line – Tpulse =20 Treset (dead-time Gaussian distribution)

D. Joint Expression for Small Non-linearity

If we take Taylor series of (13) limited to 3-order term assuming small nonlinearity of output signal it results in linear approximation of *Fnl* similar to (17).

It may be useful also to construct joint solution for small non-linearity of arbitrary *Tpulse*. All options are given below:

$$Fnl = 1 + \frac{Nph}{Nsat} \tag{18}$$

where

$$Nsat \begin{cases} \cong \dfrac{2 \cdot Npix}{PDE}, & Treset > Tpulse \\ = \dfrac{Npix}{PDE} \cdot \dfrac{Tpulse}{Treset}, & Treset << Tpulse \\ \approx \dfrac{2 \cdot Npix}{PDE} \cdot \left(1 + \dfrac{Tpulse}{2 \cdot Treset}\right), & \forall Tpulse \end{cases} \tag{19}$$

V. Efficiency of SSPM in Photon Number Resolution

A. Photon Number Resolution

PNR expressions above are just presentation of nonlinear behavior of resolution, namely its significant increase at high signal level. To complete *PNR* we have to include terms responsible for dark noise, noise of avalanche multiplication, cross-talk and afterpulsing (noise of electronics may be neglected for SSPM with high gain as well as for PMT). Combining all terms we collect all key factors in final expression:

$$PNR = \sqrt{\frac{1}{Nph}} \cdot \sqrt{\frac{Fm \cdot Fdup \cdot Fdark \cdot Fnl}{PDE}} \tag{20}$$

Where Fnl may be expressed in form (13) and (17), or (18) and (19), and dark term may be rewritten in form:

$$Fdark = 1 + \frac{Ndark}{Nph}; \quad Ndark = \frac{Nd}{PDE} \tag{21}$$

B. Excess Noise Factor and Detective Quantum Efficiency

On our opinion, it seems to be reasonable to interpret (20) as a product of ideal detector resolution equal to resolution of incident photon signal *(1/√Nph)* to a set of “excess noise factors” corresponding to different sources of noise.

Using this approach we may represent intrinsic properties of detector in form of total excess noise factor of detection:

$$ENF = Fpde \cdot Fm \cdot Fdup \cdot Fdark \cdot Fnl \tag{22}$$

Here we substitute denominator term from (20) by excess noise factor of single photon detection *Fpde* because it may be directly derived from Bernoulli distribution law of random detection process where probability of success is *PDE*:

$$Fpde = \frac{1}{PDE}; \qquad 1 + \frac{\sigma_{pde}^{\;2}}{\mu_{pde}^{\;2}} = 1 + \frac{PDE \cdot (1 - PDE)}{PDE^2} = \frac{1}{PDE} \tag{23}$$

Inversely, *1/ENF* may be interpreted as total efficiency of detection in sense of *PNR*, namely Detective Quantum Efficiency (*DQE*).

$$DQE = \frac{1}{ENF} \tag{23}$$

Both expressions (22) and (23) allow separately rank all factors resulting in degradation of *PNR* in comparative way.

Let us summarize all of them from point of view of information losses in the detection process:

1) *Fpde* - losses of single photon hits in active pixel,
2) *Fm* - fluctuation of gain in multiplied signal,
3) *Fdark* - dark noise with dissipation of signal,
4) *Fdup* - fluctuation of duplication events,
5) *Fnl* - losses of signals in already fired or dead pixels.

VI. Examples of Model Results

A. Model Results Representation

Let us apply model results to some SSPM using representation in form of *ENF* and *DQE* (Fig. 5).

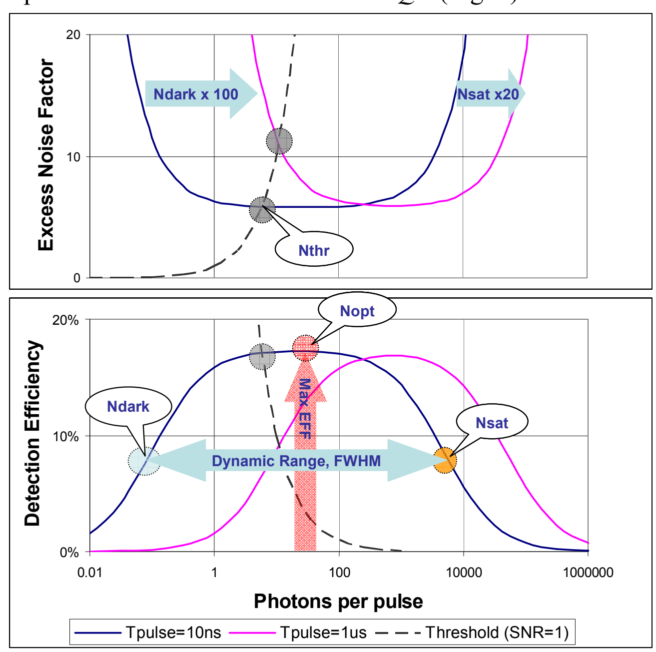

Fig. 5. ENF and DQE of SSPM with PDE=20%, Pdup=15%, DCR=1Mcps, Treset=50ns, Npix=500;
Blue solid line - for short light pulse (Tpulse = Tgate = 10 ns),
Pink solid line - for long light pulse (Tpulse = Tgate = 1 us),
Grey dot line - for determining threshold sensitivity point Nthr in photons.

Fig. 5 shows where SSPM performance is limited due to dark noise at the level of Ndark for low number of photons and due to saturation at the level of Nsat, so as actual dynamic range DR may be defined as FWHM of DQE between these points in logarithmic scale.

$$DR = Nsat/Ndark \quad (24)$$

Inside the dynamic range SSPM performance has more or less expressed plateau with optimal point *Nopt:*

$$Nopt = \sqrt{Ndark \cdot Nsat} \quad (25)$$

Threshold sensitivity of SSPM, i.e. minimal detectable number of photons *Nthr* corresponding to SNR_out = 1 is determined by equation:

$$ENF(Nph)=Nph \ @ \ Nph=Nthr. \quad (26)$$

B. Application Case Study

The example of application case study presented below (Fig. 6) is imaginary "Medical Imaging" as well as SSPMs with the alias "MPPC" (Hamamatsu MPPCs are widely used series of detectors with well-known parameters). This example is exclusively focused on demonstration of relative influence of SSPM parameters (Table 1) on *DQE* predicted by the model with some rough correspondence with well-known trends and trade-offs in SSPM design.

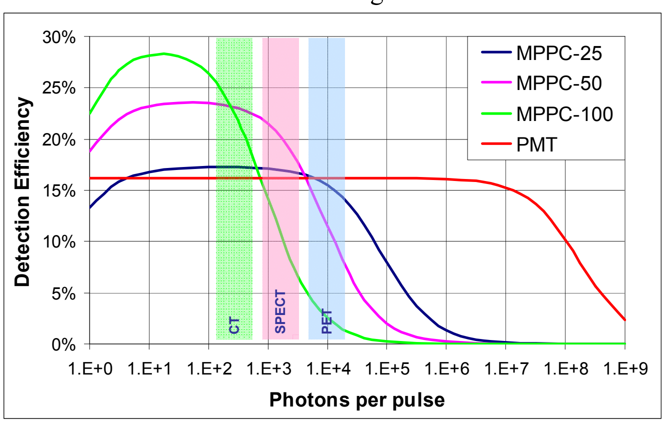

Fig. 6. Medical Imaging applications case study (100 ns detection time)

TABLE 1. ASSUMED PARAMETERS FOR THE DQE MODEL (FIG. 6)

SSPM (Fm=1.05)	Npix	PDE	Pdup	DCR, cps	Treset, s
"MPPC-25"	1600	20%	10%	3.0E+05	1.0E-08
"MPPC-50"	400	30%	20%	4.0E+05	1.5E-08
"MPPC-100"	100	40%	30%	6.0E+05	3.0E-08
"PMT" (Fm=1.2)	1.0E+06	20%	1%	1.0E+02	3.0E-09

C. DQE mapping example

In the same way the model may be used in two-dimensional mapping of *DQE (ENF, PNR)* versus number of photons and detection time or pulse duration time. Fig. 7 shows how binomial saturation is transformed into count rate saturation (left upper boundary from ~ 10^4 photons, 10^{-8} s to ~ 10^5 photons, $3 \cdot 10^{-6}$ s), how shot noise limited threshold is transformed into dark noise limited one (left lower boundary from ~ 6 photons, 10^{-7} s to ~ 30 photons, 10^{-5} s), and where are the best applicability area of the device.

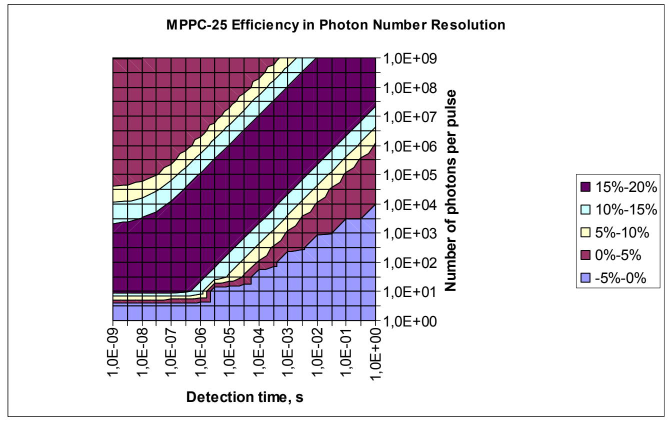

Fig. 7. Two-dimensional DQE map for "MPPC-25". Negative values (blue) represent area where signal is lower then threshold sensitivity.

VII. DISCUSSION

Proposed analytical model allows to find rough but very fast and clear answers on the questions like these: how to select the best detector for specific application in sense of *PNR* or how to optimize existing detector. Indeed, if in particular application number of photons per pulse and pulse duration time are fixed and known apriori, then we need to achieve maximum of *DQE* at this condition (some point on Fig. 7 map):

$$DQE_{\max} \sim \frac{PDE}{(1+Pdup)} \quad @ Nopt \sim Area \cdot \sqrt{n_{dcr} \cdot n_{pix}} \quad (27)$$

Where n_{dcr} and n_{pix} are corresponding densities per Area.

Expression (27) means if this condition is within dynamic range (the flat plateau of *DQE*), then the most important is to achieve maximum of *PDE* to *Fdup* ratio (*Fdark* and *Fnl* are negligible on plateau, and *Fm* is assumed to be negligible in any case). The best fit happens at *Nph=Nopt*, however *Nopt* may be adjusted by selecting different area of SSPM (if possible).

The most challenging case is if we need to optimize resolution in wide dynamic range:

$$(DQE \cdot DR)_{\max} \sim \frac{PDE \cdot n_{pix}}{(1+Pdup) \cdot dcr \cdot T_{res}} \quad (28)$$

Where *dcr* is density of *DCR*.

We assume that space for improvement of *Pdup, dcr*, and *Treset* is rather limited at present SSPM design and technology level. In the same time in order to make simultaneous improvement of *PDE* and density of pixels one have to resolve existing trade-off between these parameters in GM APD matrix design. It is very challenging and very promising goal. In our opinion SSPM could outperform PMT as universal detector for wide application areas only if this goal will be achieved.

VIII. Conclusion

Analytical model of Photon Number Resolution as well as total Excess Noise Factor and Detective Quantum Efficiency was developed taking into account very limited number of key factors and parameters of SSPM. The model is based on probabilistic approach of reasonable simplicity and shows reasonable results. However it should be validated in experiment.

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